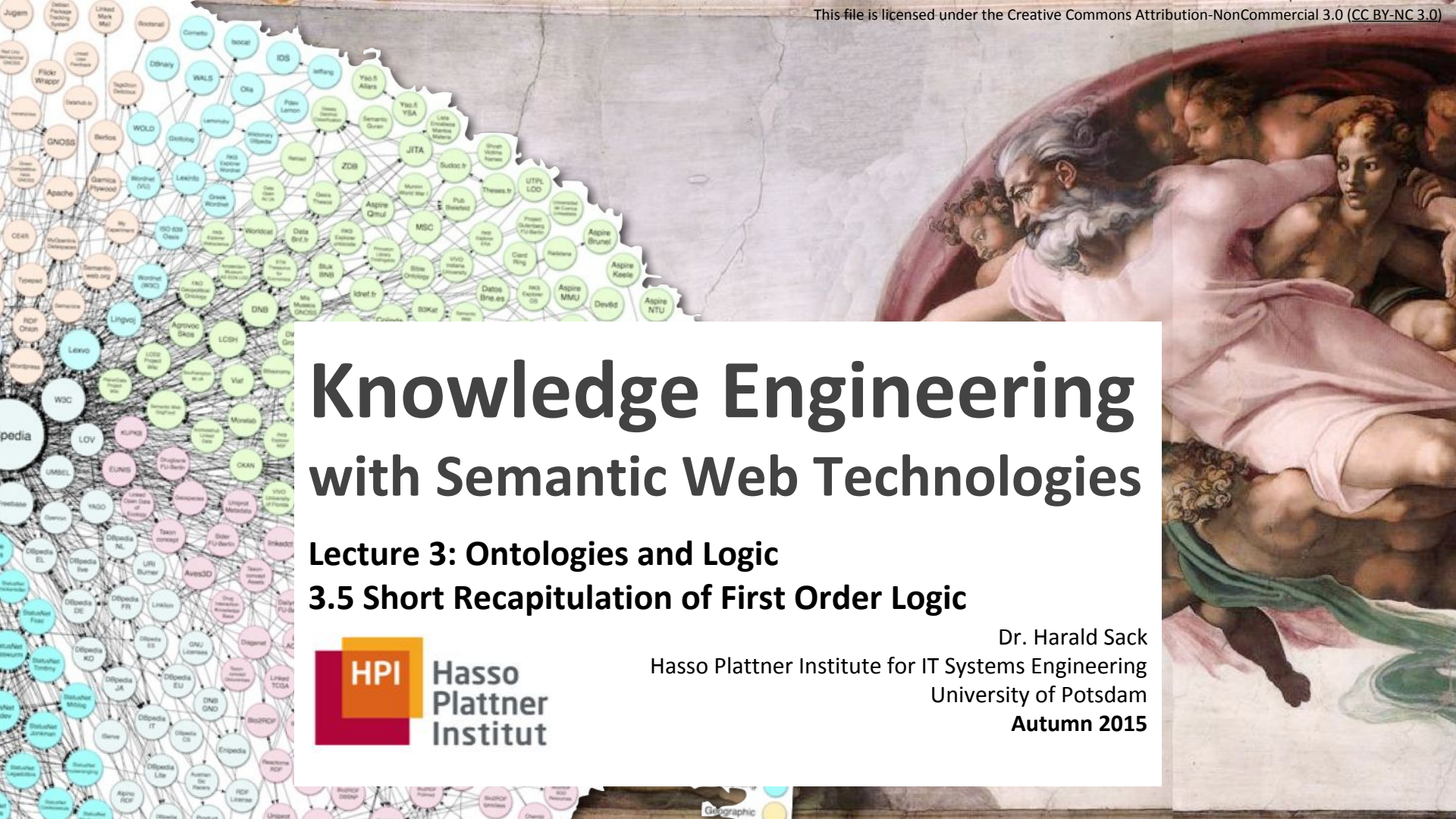


The image is a composite. On the left, there is a large, dense network graph with numerous circular nodes in various colors (blue, green, orange, pink) connected by thin lines. Some nodes are labeled with text like 'Wikipedia', 'LOV', 'HPI', 'Hasso Plattner Institut', 'Geographic', 'HPI', 'Hasso Plattner Institut', 'Geographic', 'HPI', 'Hasso Plattner Institut', 'Geographic'. On the right, there is a classical painting of a man with a long white beard and hair, wearing a pink robe, looking down at a woman who is reclining. The man's hand is resting on the woman's shoulder. The background of the painting is a soft, hazy landscape. In the center, there is a white rectangular box with black text. The text reads: 'Knowledge Engineering with Semantic Web Technologies', 'Lecture 3: Ontologies and Logic', '3.5 Short Recapitulation of First Order Logic', and 'Dr. Harald Sack'. Below the text is the logo of the Hasso Plattner Institut, which consists of a red square with the letters 'HPI' in white, followed by the text 'Hasso Plattner Institut' in black. At the top right of the image, there is a small text line: 'This file is licensed under the Creative Commons Attribution-NonCommercial 3.0 (CC BY-NC 3.0)'.

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First Order Logic - FOL

quantifier	name	intentional meaning
$\exists$	existential quantifier	"it exists"
$\forall$	universal quantifier	"for all"

- **Operators (logical connectives)** as in propositional logic
- **Variables**, e.g., $X, Y, Z, \dots$
- **Constants**, e.g., $a, b, c, \dots$
- **Functions**, e.g., $f, g, h, \dots$ (incl. arity)
- **Relations / Predicates**, e.g., $p, q, r, \dots$ (incl. arity)

$$(\forall X)(\exists Y)((p(X) \vee \neg q(f(X), Y)) \rightarrow r(X))$$

First Order Logic - FOL

FOL: Syntax

- „correct“ formulation of **Terms** from Variables, Constants and Functions:
 - $f(X)$, $g(a, f(Y))$, $s(a)$, $.(H, T)$, $x_location(Pixel)$
- „correct“ formulation of **Atoms** from Relations with **Terms** as arguments
 - $p(f(X))$, $q(s(a), g(a, f(Y)))$, $add(a, s(a), s(a))$,
 $greater_than(x_location(Pixel), 128)$
- „correct“ formulation of **Formulas** from **Atoms**, Operators and Quantifiers:
 - $(\forall Pixel)(greater_than(x_location(Pixel), 128) \rightarrow red(Pixel))$
- If in doubt, use brackets!
- All Variables should be quantified!

First Order Logic - FOL

How to model Facts?

- „All kids love icecream.“
 $\forall X: \text{Child}(X) \rightarrow \text{lovesIcecream}(X)$
- „The father of a person is its male parent.“
 $\forall X \forall Y: \text{isFather}(X,Y) \leftrightarrow (\text{Male}(X) \wedge \text{isParent}(X,Y))$
- „There are (one or more) interesting lectures.“
 $\exists X: \text{Lecture}(X) \wedge \text{Interesting}(X)$
- „The relation ,isNeighbor‘ is symmetric.“
 $\forall X \forall Y: \text{isNeighbor}(X,Y) \rightarrow \text{isNeighbor}(Y,X)$

First Order Logic - FOL

How to model Facts?

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First Order Logic - FOL

How to model Facts?

- There is a significant difference:

- „All kids love icecream.“

$\forall X: \text{Child}(X) \rightarrow \text{lovesIcecream}(X)$

it is possible that X is not a kid but (nevertheless) loves icecream

$\forall X: \text{Child}(X) \wedge \text{lovesIcecream}(X)$

there are only kids in the universe of discourse and they love icecream

$\forall X: \text{Child}(X)$	$\forall X: \text{lovesIcecream}(x)$	$\forall X: \text{Child}(X) \rightarrow \text{lovesIcecream}(X)$	$\forall X: \text{Child}(X) \wedge \text{lovesIcecream}(X)$
0	0	1	0
0	1	1	0
1	0	0	0
1	1	1	1

First Order Logic - FOL

How to model Facts?

- There is a significant difference:

- „There are (one or more) interesting lectures.“

$\exists X: \text{Lecture}(X) \wedge \text{Interesting}(X)$

the things we address here must have both properties

$\exists X: \text{Lecture}(X) \rightarrow \text{Interesting}(X)$

here X is also qualified, if X is not a lecture

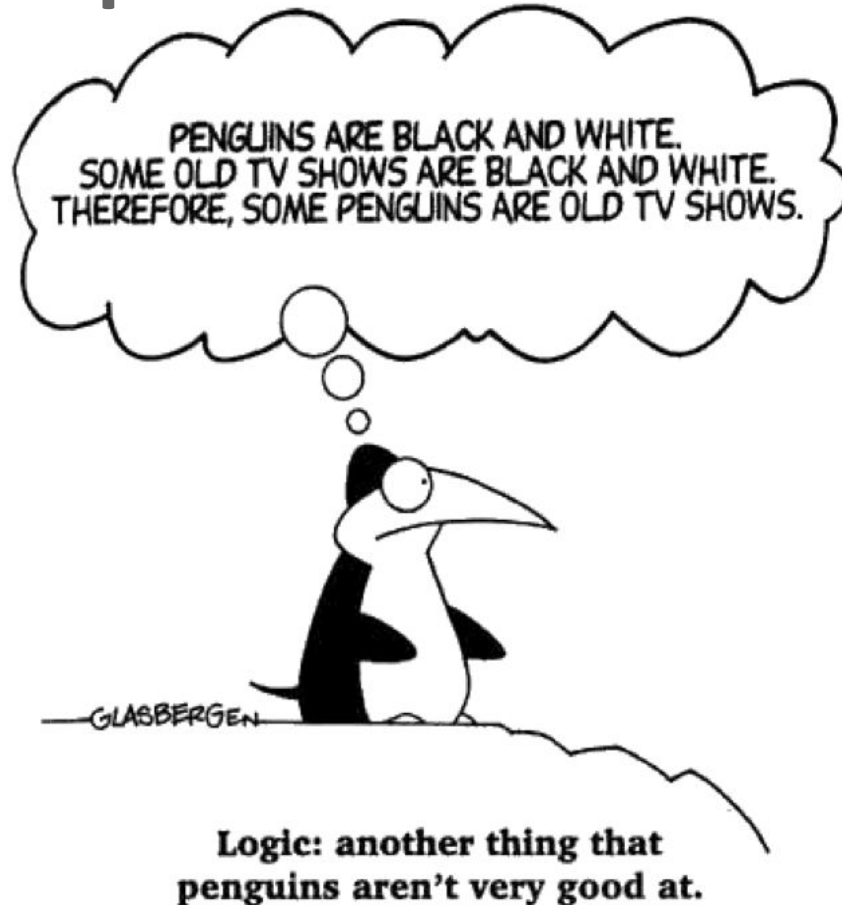
$\exists X: \text{Lecture}(X)$	$\exists X: \text{Interesting}(X)$	$\exists X: \text{Lecture}(X) \rightarrow \text{Interesting}(X)$	$\exists X: \text{Lecture}(X) \wedge \text{Interesting}(X)$
0	0	1	0
0	1	1	0
1	0	0	0
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First Order Logic - FOL

Model-theoretic Semantics

- **Structure:**
 - Definition of a **domain** D.
 - **Constant symbols** are mapped to elements of D.
 - **Function symbols** are mapped to functions in D.
 - **Relation symbols** are mapped to relations over D.
- **Then:**
 - **Assertions** will become elements of D.
 - **Relation symbols** with arguments will become **true** or **false**.
 - **Logical connectives** and **quantifiers** are treated likewise.

The Penguin Example



The Penguin Example

```
( (∀X)( penguin(X) → blackandwhite(X) )  
∧ (∃X)( oldTVshow(X) ∧ blackandwhite(X)  
)  
 ) → (∃X)( penguin(X) ∧ oldTVshow(X) )
```

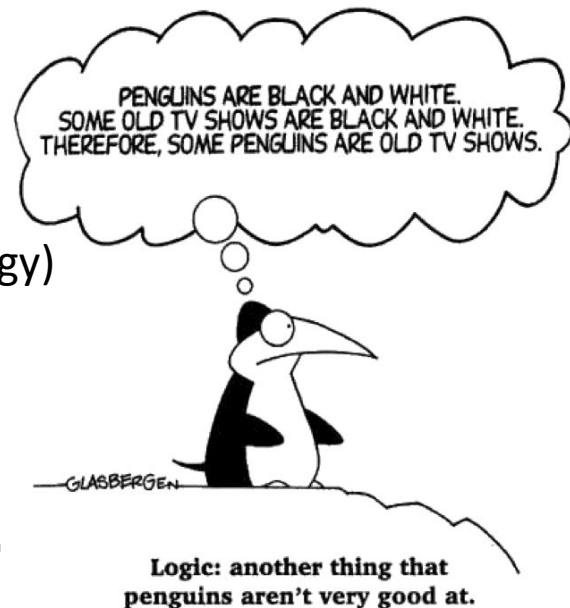
What is the intentional Semantics?



The Penguin Example

$$\begin{aligned} & ((\forall X)(\text{penguin}(X) \rightarrow \text{blackandwhite}(X)) \\ & \wedge (\exists X)(\text{oldTVshow}(X) \wedge \text{blackandwhite}(X)) \\ &) \\ &) \rightarrow (\exists X)(\text{penguin}(X) \wedge \text{oldTVshow}(X)) \end{aligned}$$

- Interpretation $\mathcal{I}$:
 - **Domain:** a set M , containing elements a, b .
 - ... no constants or function symbols ...
 - We show: the formula is *refutable* (i.e. it is not a tautology)
 - If $\mathcal{I}(\text{penguin})(a)$, $\mathcal{I}(\text{blackandwhite})(a)$,
 $\mathcal{I}(\text{oldTVshow})(b)$, $\mathcal{I}(\text{blackandwhite})(b)$ is true,
 $\mathcal{I}(\text{penguin})(b)$ and $\mathcal{I}(\text{oldTVshow})(a)$ is false,
 - then the formula with Interpretation $\mathcal{I}$ is **false**, i.e. $\mathcal{I} \not\models \mathcal{F}$



First Order Logic - FOL

Logical Entailment

- a **theory** $\mathbb{T}$ is a set of formulas.
- an interpretation $\mathbb{I}$ is a **model** of $\mathbb{T}$, iff $\mathbb{I} \models G$ for all formulas G in $\mathbb{T}$.
- a formula F is a **logical consequence** of $\mathbb{T}$,
iff all models of $\mathbb{T}$ are also models of F .
- then we write $\mathbb{T} \models F$.
- two formulas F, G are called **logically equivalent**,
iff $\{F\} \models G$ and $\{G\} \models F$.
- then we write $F \equiv G$

Theory $\hat{=}$ Knowledge Base

First Order Logic - FOL

Some Properties of FOL

- **Monotony**

- If the knowledge base grows, all previously possible entailments hold.
- S and T are theories, with $S \subseteq T$
- Then it holds that $\{F \mid S \models F\} \subseteq \{F \mid T \models F\}$

- **Compactness**

- For each entailment made from a theory, a finite subset of the theory is sufficient.

First Order Logic - FOL

Some Properties of FOL

- **Semi-decidability**

- FOL is **not decidable**
- But, FOL is **semi-decidable**,
i.e. a logical consequence $\mathcal{T} \models \mathcal{F}$ always can be
proven in finite time (but not necessarily also $\mathcal{T} \not\models \mathcal{F}$)



06: How to Mechanize Reasoning - Tableaux Algorithm

OpenHPI - Course Knowledge Engineering with Semantic Web Technologies

Lecture 3: Ontologies and Logic